1. (a) Simplify fully

$$\frac{2x^2+9x-5}{x^2+2x-15}$$

(3)

Given that

$$\ln(2x^2+9x-5) = 1 + \ln(x^2+2x-15), \quad x \neq -5,$$

(b) find x in terms of e.

(4)
(Total 7 marks)

2. Express

$$\frac{x+1}{3x^2-3} - \frac{1}{3x+1}$$

as a single fraction in its simplest form.

(Total 4 marks)

3. The function f is defined by

$$\mathrm{f}(x) = 1 - \frac{2}{(x+4)} + \frac{x-8}{(x-2)(x+4)}, \quad x \in \mathbb{R}, x \neq -4, x \neq 2$$

(a) Show that $\mathrm{f}(x) = \dfrac{x-3}{x-2}$

(5)

The function g is defined by

$$g(x) = \frac{\mathrm{e}^x - 3}{\mathrm{e}^x - 2}, \qquad x \in \mathbb{R}, x \neq \ln 2$$

(b) Differentiate g(x) to show that $\mathrm{g}'(x) = \dfrac{\mathrm{e}^x}{(\mathrm{e}^x - 2)^2}$,

(3)

(c) Find the exact values of x for which $\mathrm{g}'(x) = 1$

(4)
(Total 12 marks)

4. $$\mathrm{f}(x) = \frac{2x+2}{x^2-2x-3} - \frac{x+1}{x-3}$$

(a) Express f (x) as a single fraction in its simplest form.

(4)

(b) Hence show that $\mathrm{f}'(x) = \dfrac{2}{(x-3)^2}$

(3)
(Total 7 marks)

5. Given that

$$\frac{2x^4 - 3x^2 + x + 1}{(x^2-1)} \equiv (ax^2 + bx + c) + \frac{dx+e}{(x^2-1)},$$

find the values of the constants a, b, c, d and e.

(Total 4 marks)

6. $$\mathrm{f}(x) = 1 - \frac{3}{\mathrm{x}+2} + \frac{3}{(x+2)^2}, x \neq -2.$$

(a) Show that $\mathrm{f}(x) = \dfrac{x^2+x+1}{(x+2)^2}, x \neq -2.$

(4)

(b) Show that $x^2 + x + 1 > 0$ for all values of x.

(3)

(c) Show that f (x) > 0 for all values of x, $x \neq -2$.

(1)
(Total 8 marks)

7. (a) Simplify $\dfrac{3x^2 - x - 2}{x^2 - 1}$

(3)

(b) Hence, or otherwise, express $\dfrac{3x^2 - x - 2}{x^2 - 1} - \dfrac{1}{x(x+1)}$ as a single fraction in its simplest form.

(3)
(Total 6 marks)

8. Express

$$\frac{2x^2 + 3x}{(2x+3)(x-2)} - \frac{6}{x^2 - x - 2}$$

as a single fraction in its simplest form.

(Total 7 marks)

9. The function f is defined by

f: $x \to \dfrac{5x+1}{x^2 + x - 2} - \dfrac{3}{x+2}$, $x > 1$.

(a) Show that f(x) = $\dfrac{2}{x-1}$, $x > 1$.

(4)

(b) Find $f^{-1}(x)$.

(3)

The function g is defined by

$$g: x \to x^2 + 5, \quad x \in \mathbb{R}.$$

(c) Solve fg(x) = $\frac{1}{4}$.

(3)
(Total 10 marks)

10. $$\mathrm{f}(x) = \frac{x^2 - x - 6}{x^2 - 3x}, \quad x \neq 0, \quad x \neq 3.$$

(a) Express f(x) in its simplest form.

(3)

(b) Hence, or otherwise, find the exact solutions of f(x) = x + 1.

(3)
(Total 6 marks)

11.

$$\mathrm{f}(x) = \frac{2x + 5}{x + 3} - \frac{1}{(x + 3)(x + 2)}, \quad x > -2.$$

(a) Express f(x) as a single fraction in its simplest form.

(5)

(b) Hence show that $\mathrm{f}(x) = 2 - \dfrac{1}{x + 2}, \quad x > -2.$

(2)

The curve $y = \dfrac{1}{x}$, $x > 0$, is mapped onto the curve y = f(x), using three successive transformations T_1, T_2 and T_3, where T_1 and T_3 are translations.

(c) Describe fully T_1, T_2 and T_3.

(4)
(Total 11 marks)

12. Express as a single fraction in its simplest form

$$\frac{x^2 - 8x + 15}{x^2 - 9} \times \frac{2x^2 + 6x}{(x - 5)^2}.$$

(Total 4 marks)

13. The function f is given by

$$\mathrm{f} : x \mapsto 2 + \frac{3}{x+2}, \quad x \in \mathbb{R}, \quad x \neq -2.$$

(a) Express $2 + \dfrac{3}{x+2}$ as a single fraction.

(1)

(b) Find an expression for $\mathrm{f}^{-1}(x)$.

(3)

(c) Write down the domain of f^{-1}.

(1)
(Total 5 marks)

14. (a) Express as a fraction in its simplest form

$$\frac{2}{x-3} + \frac{13}{x^2 + 4x - 21}.$$

(3)

(b) Hence solve

$$\frac{2}{x-3} + \frac{13}{x^2 + 4x - 21} = 1.$$

(3)
(Total 6 marks)

15. (a) Simplify $\dfrac{x^2 + 4x + 3}{x^2 + x}$.

(2)

(b) Find the value of x for which $\log_2 (x^2 + 4x + 3) - \log_2 (x^2 + x) = 4$.

(4)
(Total 6 marks)

16. Express $\dfrac{x}{(x+1)(x+3)} + \dfrac{x+12}{x^2-9}$ as a single fraction in its simplest form.

(Total 6 marks)

17. Express

$$\frac{3x^2}{(2x^2+7x+6)} \times \frac{7(3+2x)}{3x^5}$$

as a single fraction in its simplest form.

(Total 4 marks)

1. (a) $\dfrac{(x+5)(2x-1)}{(x+5)(x-3)} = \dfrac{(2x-1)}{(x-3)}$ M1 B1 A1 aef 3

Note

M1: An attempt to factorise the numerator.
B1: Correct factorisation of denominator to give $(x + 5)(x - 3)$.
Can be seen anywhere.

(b) $\ln\left(\dfrac{2x^2+9x-5}{x^2+2x-15}\right)=1$ M1

$\dfrac{2x^2+9x-5}{x^2+2x-15}=\mathrm{e}$ dM1

$\dfrac{2x-1}{x-3}=\mathrm{e} \Rightarrow \quad 3\mathrm{e}-1 = x(\mathrm{e}-2)$ M1

$\Rightarrow \; x = \dfrac{3\mathrm{e}-1}{\mathrm{e}-2}$ A1 aef cso 4

Note

M1: Uses a correct law of logarithms to combine at least two terms.
This usually is achieved by the subtraction law of logarithms to give
$\ln\left(\dfrac{2x^2+9x-5}{x^2+2x-15}\right)=1$
The product law of logarithms can be used to achieve
$\ln(2x^2 + 9x - 5) = \ln(\mathrm{e}(x^2 + 2x - 15))$.
The product and quotient law could also be used to achieve
$\ln\left(\dfrac{2x^2+9x-5}{e(x^2+2x-15)}\right)=0$

dM1: Removing ln's correctly by the realisation that the anti-ln of 1 is e.
Note that this mark is dependent on the previous method mark being awarded.
M1: Collect x terms together and factorise.
Note that this is not a dependent method mark.

A1: $\dfrac{3e-1}{e-2}$ or $\dfrac{3e^1-1}{e^1-2}$ or $\dfrac{1-3e}{2-e}$. **aef**

Note that the answer needs to be in terms of e. The decimal answer is 9.9610559…

Note that the solution must be correct in order for you to award this final accuracy mark.

[7]

2. $\dfrac{x+1}{3x^2-3} - \dfrac{1}{3x+1}$

$= \dfrac{x+1}{3(x^2-3)} - \dfrac{1}{3x+1}$

$x^2 - 1 \rightarrow (x+1)(x-1)$ or

$3x^2 - 3 \rightarrow (x+1)(3x-3)$ or **Award**

$\dfrac{x+1}{3(x+1)(x-1)} - \dfrac{1}{3x+1}$ $3x^2 - 3 \rightarrow (3x+3)(x-1)$ **below**

seen or implied anywhere in candidate's working.

$= \dfrac{1}{3(x-1)} - \dfrac{1}{3x+1}$

$= \dfrac{3x+1-3(x-1)}{3(x-1)(3x+1)}$ Attempt to combine. M1

or $\dfrac{3x+1}{3(x-1)(3x+1)} - \dfrac{3(x-1)}{3(x-1)(3x+1)}$ Correct result. A1

Decide to award M1 here!! **M1**

Either $\dfrac{4}{3(x-1)(3x+1)}$

$= \dfrac{4}{3(x-1)(3x+1)}$ or $\dfrac{\frac{4}{3}}{(x-1)(3x+1)}$ or $\dfrac{4}{(3x-3)(3x+1)}$ A1 **aef**

or $\dfrac{4}{9x^2-6x-3}$

[4]

3. (a) $f(x) = 1 - \dfrac{2}{(x+4)} + \dfrac{x-8}{(x-2)(x+4)}$

$x \in \mathbb{R}, x \neq -4, x \neq 2.$

$f(x) = \dfrac{(x-2)(x+4) - 2(x-2) + x - 8}{(x-2)(x+4)}$ An attempt to combine to one fraction M1

Correct result of combining all three fractions A1

$= \dfrac{x^2 + 2x - 8 - 2x + 4 + x - 8}{(x-2)(x+4)}$

$= \dfrac{x^2 + x - 12}{[(x+4)(x-2)]}$ Simplifies to give the correct

	numerator. Ignore omission of denominator	A1	
$=\dfrac{(x+4)(x-3)}{[(x+4)(x-2)]}$	An attempt to factorise the numerator.	dM1	
$=\dfrac{(x-3)}{(x-2)}$	Correct result	A1 cso **AG**	5

(b) $g(x) = \dfrac{e^x-3}{e^x-2} \quad x \in \mathbb{R}, x \neq \ln 2.$

Apply quotient rule: $\left\{\begin{matrix} u = e^x - 3 & v = e^x - 2 \\ \dfrac{du}{dx} = e^x & \dfrac{dv}{dx} = e^x \end{matrix}\right\}$

$g'(x) = \dfrac{e^x(e^x-2) - e^x(e^x-3)}{(e^x-2)^2}$	Applying $\dfrac{vu'-uv'}{v^2}$	M1	
	Correct differentiation	A1	
$=\dfrac{e^{2x}-2e^x-e^{2x}+3e^x}{(e^x-2)^2}$			
$=\dfrac{e^x}{(e^x-2)^2}$	Correct result	A1 **AG** cso	3

(c) $g'(x) = 1 \Rightarrow = \dfrac{e^x}{(e^x-2)^2} = 1$

$e^x = (e^x - 2)^2$	Puts their differentiated numerator equal to their denominator.	M1	
$e^x = e^{2x} - 2e^x - 2e^x + 4$			
$\underline{e^{2x} - 5e^x + 4} = 0$	$\underline{e^{2x} - 5e^x + 4}$	A1	
$(e^x - 4)(e^x - 1) = 0$	Attempt to factorise or solve quadratic in e^x	M1	
$e^x = 4$ or $e^x = 1$			
$x = \ln 4$ or $x = 0$	both $x = 0, \ln 4$	A1	4

[12]

4. (a) $\dfrac{2x+2}{x^2-2x-3}-\dfrac{x+1}{x-3}=\dfrac{2x+2}{(x-3)(x+1)}-\dfrac{x+1}{x-3}$

$=\dfrac{2x+2-(x+1)(x+1)}{(x-3)(x+1)}$ M1 A1

$=\dfrac{(x+1)(1-x)}{(x-3)(x+1)}$ M1

$\dfrac{1-x}{x-3}$ Accept $-\dfrac{x-1}{x-3},\dfrac{x-1}{3-x}$ A1 4

Alternative

$\dfrac{2x+2}{x^2-2x-3}=\dfrac{2(x+1)}{(x-3)(x+1)}=\dfrac{2}{x-3}$ M1 A1

$\dfrac{2}{x-3}-\dfrac{x+1}{x-3}=\dfrac{2-(x+1)}{x-3}$ M1

$\dfrac{1-x}{x-3}$ A1 4

(b) $\dfrac{d}{dx}\left(\dfrac{1-x}{x-3}\right)=\dfrac{(x-3)(-1)-(1-x)1}{(x-3)^2}$ M1 A1

$=\dfrac{-x+3-1+x}{(x-3)^2}=\dfrac{2}{(x-3)^2}$ * cso A1 3

Alternatives

1 f(x) = $\dfrac{1-x}{x-3}=-1-\dfrac{2}{x-3}=-1-2(x-3)^{-1}$

f'(x) = $(-1)(-2)(x-3)^{-2}$ M1 A1

$=\dfrac{2}{(x-3)^2}$ * cso A1 3

2 f(x) = $(1-x)(x-3)^{-1}$

f'(x) = $(-1)(x-3)^{-1}+(1-x)(-1)(x-3)^{-2}$ M1

$=-\dfrac{1}{x-3}-\dfrac{1-x}{(x-3)^2}=\dfrac{-(x-3)-(1-x)}{(x-3)^2}$ A1

$=\dfrac{2}{(x-3)^2}$ * A1 3

[7]

5.

$$\begin{array}{r|l} & 2x^2 \quad -1 \\ \hline x^2-1 & 2x^4 \quad -3x^2+x+1 \\ & 2x^4 \quad -2x^2 \\ \hline & \quad -x^2+x+1 \\ & \quad -x^2 \quad\; +1 \\ \hline & \qquad x \end{array}$$

	M1
$a = 2$ stated or implied	A1
$c = -1$ stated or implied	A1

$$2x^2 - 1 + \frac{x}{x^2-1}$$

$a = 2$, $b = 0$, $c = -1$, $d = 1$, $e = 0$

$d = 1$ and $b = 0$, $e = 0$ stated or implied — A1

[4]

6. (a) $f(x) = \dfrac{(x+2)^2, -3(x+2)+3}{(x+2)^2}$ — M1A1, A1

$$= \frac{x^2+4x+4-3x-6+3}{(x+2)^2} = \frac{x^2+x+1}{(x+2)^2}*$$ cso — A1 — 4

(b) $x^2 + x + 1 = \left(x+\frac{1}{2}\right)^2 + \frac{3}{4}, > 0$ for all values of x. — M1A1, A1 — 3

Alternative

$\dfrac{d}{dx}(x^2+x+1) = 2x+1 = 0 \Rightarrow x = -\dfrac{1}{2} \Rightarrow x^2+x+1 = \dfrac{3}{4}$ — M1A1

A parabola with positive coefficient of x^2 has a minimum

$\Rightarrow x^2 + x + 1 > 0$

Accept equivalent arguments — A1 — 3

(c) $f(x) = \dfrac{\left(x+\frac{1}{2}\right)^2 + \frac{3}{4}}{(x+2)^2}$

Numerator is positive from (b)

$x \neq -2 \Rightarrow (x + 2)^2 > 0$ (Denominator is positive)

Hence $f(x) > 0$ — B1 — 1

[8]

7.	(a)	$\frac{(3x-2)(x-1)}{(x+1)(x-1)}$, $\frac{3x+2}{x+1}$	M1B1, A1	3
		M1 attempt to factorise numerator, usual rules *B1 factorising denominator seen anywhere in (a),* *A1 given answer* *If factorisation of denom. not seen, correct answer implies B1*		
	(b)	Expressing over common denominator		
		$\frac{3x+2}{x+1}-\frac{1}{x(x+1)}=\frac{x(3x+2)-1}{x(x+1)}$	M1	
		[Or "Otherwise": $\frac{(3x^2-x-2)x-(x-1)}{x(x^2-1)}$]		
		Multiplying out numerator and attempt to factorise	M1	
		[$3x^2+2x-1 \equiv (3x-1)(x+1)$]		
		$\frac{3x-1}{x}$	Al	3

[6

8.	$\frac{x(2x+3)}{(2x+3)(x-2)}-\frac{6}{(x-2)(x+1)}$	B1, B1	
	$=\frac{x(x-1)-6}{(x-2)(x+1)}$	M1 A1ft	
	$=\frac{(x+3)(x-2)}{(x-2)(x+1)}$	M1 A1	
	$=\frac{x+3}{x+1}$	A1	7

Alternative 1:

$\frac{2x^2+3x}{(2x+3)(x-2)}-\frac{6}{(x-2)(x+1)}$	B1
$=\frac{(2x^2+3x)(x+1)-6(2x+3)}{(2x+3)(x-2)(x+1)}$	M1 A1ft
$=\frac{(2x^3+5x^2-9x-18)}{(2x+3)(x-2)(x+1)}$	A1

$= \dfrac{(x-2)(2x^2+9x+9)}{(2x+3)(x-2)(x+1)}$ M1

$= \dfrac{(x-2)(2x+3)(x+3)}{(2x+3)(x-2)(x+1)}, = \dfrac{x+3}{x+1}$ A1, A1

Alternative 2:

$\dfrac{2x^2+3x}{(2x+3)(x-2)} - \dfrac{6}{(x^2-x-2)}$

$= \dfrac{x(2x+3x)}{(2x+3)(x-2)} - \dfrac{6}{x^2-x-2}$ B1

$= \dfrac{x(x^2-x-2)-6(x-2)}{(x-2)(x^2-x-2)}, = \dfrac{x^3-x^2-2x-6x+12}{(x-2)(x^2-x-2)}$ M1 A1ft

$= \dfrac{x^3-x^2-8x+12}{(x-2)(x^2-x-2)}$ A1

$= \dfrac{(x-2)(x^2+x-6)}{(x-2)(x^2-x-2)}$ M1

$= \dfrac{(x+3)(x-2)}{(x-2)(x+1)}, = \dfrac{x+3}{x+1}$ A1, A1

[7]

9. (a) $f(x) = \dfrac{5x+1}{(x+2)(x-1)} - \dfrac{3}{x+2}$ B1

factors of quadratic denominator

$= \dfrac{5x+1-3(x-1)}{(x+2)(x-1)}$

common denominator M1
simplifying to linear numerator M1

$= \dfrac{2x+4}{(x+2)(x-1)} = \dfrac{2(x+2)}{(x+2)(x-1)} = \dfrac{2}{x-1}$ AG A1cso 4

(b) $y = \dfrac{2}{x-1} \Rightarrow xy - y = 2 \Rightarrow$ M1

$xy = 2 + y$ or $x - 1 = \dfrac{2}{y}$ A1

$f^{-1}(x) = \dfrac{2+x}{x}$ or equiv. A1 3

	(c)	$\text{fg}(x) = \dfrac{2}{x^2+4}$ (attempt) $\left[\dfrac{2}{\text{"}g\text{"}-1}\right]$	M1	
		Setting $\dfrac{2}{x^2+4} = \dfrac{1}{4}$ and finding $x^2 = \ldots$; $x = \pm 2$	DM1; A1	3
				[10]

10.	(a)	$\dfrac{(x-3)(x+2)}{x(x-3)}; = \dfrac{(x+2)}{x}$ or $1+\dfrac{2}{x}$	B1,B1,B1	3
		B1 numerator, B1 denominator ; *B1 either form of answer*		
	(b)	$\dfrac{(x+2)}{x} = x+1 \Rightarrow x^2 = 2$	M1 A1ft	
		M1 for equating f(x) to x + 1 and forming quadratic. *A1 candidate's correct quadratic*		
		$x = \pm\sqrt{2}$	A1	3
				[6]

11.	(a)	$\dfrac{2x+5}{x+3} - \dfrac{1}{(x+3)(x+2)} = \dfrac{(2x+5)(x+2)-1}{(x+3)(x+2)}$	M1	
		$= \dfrac{2x^2+9x+9}{(x+3)(x+2)}$	A1	
		$= \dfrac{(2x+3)(x+3)}{(x+3)(x+2)}$	M1 A1	
		$= \dfrac{2x+3}{x+2}$	A1	5
	(b)	$2 - \dfrac{1}{x+2} = \dfrac{2(x+2)-1}{x+2} = \dfrac{2x+3}{x+2}$ or the reverse	M1 A1	2
	(c)	T_1: Translation of –2 in x direction	B1	
		T_2: Reflection in the x-axis	B1	
		T_3: Translation of (+)2 in y direction	B1	
		All three fully correct	B1	4
				[11]

One alternative is
T_1: Translation of –2 in x direction
T_2: Rotation of 90° clockwise about O
T_3: Translation of –2 in x direction

12.	$\dfrac{(x-3)(x-5)}{(x-3)(x+3)} \times \dfrac{2x(x+3)}{(x-5)^2}$ (3 × factorising)	B1 B1 B1	
	$= \dfrac{2x}{x-5}$	B1	
			[4]

13.	(a)	$2 + \dfrac{3}{x+2}\left(=\dfrac{2(x+2)+3}{x+2}\right) \quad \ddot{=} \quad \underline{\underline{\dfrac{2x+7}{x+2}}}$ or $\dfrac{2(x+2)+3}{x+2}$	B1	1

(b) $y = 2 + \dfrac{3}{x+2}$ OR $y = \dfrac{2x+7}{x+2}$

$y - 2 = \dfrac{3}{x+2}$	$y(x+2) = 2x + 7$	M1	
	$yx - 2x = 7 - 2y$		
$x + 2 = \dfrac{3}{y-2}$	$x(y-2) = 7 - 2y$	M1	
$x = \dfrac{3}{y-2} - 2$	$x = \dfrac{7-2y}{y-2}$		
$\therefore\ \underline{\underline{f^{-1}(x) = \dfrac{3}{x-2} - 2}}$	$\underline{\underline{f^{-1}(x) = \dfrac{7-2x}{x-2}}}$	o.e A1	3

Notes

M1 $y = f(x)$ and 1st step towards $x =$.

M1 One step from $x =$.

A1 y or $f^{-1}(x)$ = in terms of x.

(c)	Domain of $f^{-1}(x)$ is $x \in \mathbb{R}, x \neq 2$ [NB $x \neq +2$]	B1	1
			[5]

14.	(a)	$\dfrac{2}{x-3} + \dfrac{13}{(x-3)(x+7)}$	M1	
		$= \dfrac{2(x+7)+13}{(x-3)(x+7)} = \dfrac{2x+27}{(x-3)(x+7)}$	M1 A1	3
	(b)	$2x + 27 = x^2 + 4x - 21$	M1	
		$x^2 + 2x - 48 = (x+8)(x-6) = 0$		
		$x = -8, 6$	M1 A1	3
				[6]

15. (a) $\dfrac{x^2+4x+3}{x^2+x} = \dfrac{(x+3)(x+1)}{x(x+1)}$ M1

Attempt to factorise numerator or denominator

$= \dfrac{x+3}{x}$ or $1 + \dfrac{3}{x}$ or $(x+3)x^{-1}$ A1 2

(b) LHS = $\log_2\left(\dfrac{x^2+4x+3}{x^2+x}\right)$ M1 (*)

Use of log a – log b

RHS = 2^4 or 16 B1

$x + 3 = 16x$ M1 (*)

Linear or quadratic equation in x

(*) *dep*

$x = \frac{3}{15}$ or $\frac{1}{5}$ or 0.2 A1 4

[6]

16. $x^2 - 9 = (x-3)(x+3)$ seen B1

Attempt at forming single fraction

$\dfrac{x(x-3)+(x+12)(x+1)}{(x+1)(x+3)(x-3)}$; $= \dfrac{2x^2+10x+12}{(x+1)(x+3)(x-3)}$ M1; A1

Factorising numerator = $\dfrac{2(x+2)(x+3)}{(x+1)(x+3)(x-3)}$

or equivalent = $\dfrac{2(x+2)}{(x+1)(x-3)}$ M1 M1 A1

[6]

17. $2x^2 + 7x + 6 = (x+2)(2x+3)$ M1 A1

$\dfrac{3x^2}{(2+x)(3+2x)} \times \dfrac{7(3+2x)}{3x^5}$

$= \dfrac{7}{(2+x)x^3}$

some correct algebraic cancelling M1 A14

[4]

1. This question was well answered with candidates usually scoring either 3 marks (about 21%), or 5 marks (about 17%) or all 7 marks (about 46%).

The vast majority of candidates achieved all three marks in part (a). A significant minority of candidates used an alternative method of long division and were invariably successful in achieving the result of 2 + $\frac{5}{(x-3)}$.

The laws of logarithms caused problems for weaker candidates in part (b). Common errors including some candidates simplifying $\ln(2x^2 + 9x - 5) - \ln(x^2 + 2x - 15)$ to $\frac{\ln(2x^2+9x-5)}{\ln(x^2+2x-15)}$ or other candidates manipulating the equation $\ln(2x^2 + 9x - 5) = 1 + \ln(x^2 + 2x - 15)$ into $2x^2 + 9x - 5 = e^1 + x^2 + 2x - 15$. Perhaps more disheartening was the number of candidates who were unable to make x the subject after correctly achieving $\frac{2x-1}{x-3} = e$, with some leaving a final answer of $x = \frac{1+ex-3e}{2}$. Those candidates who used long division in part (a) usually coped better with making x the subject in part (b).

2. This question seemed to cause a problem for a significant number of candidates, with some candidates making more than one attempt at the question. The most popular method was for candidates to combine the fractions and simplify their answer to give a numerator of $4x + 4$ and a denominator of $(3x^2 - 3)(3x + 1)$. Those candidates who progressed no further only gained the first 2 marks because of a failure to factorise $3x^2 - 3$. At this stage a significant proportion of these candidates factorised both $4x + 4$ and $(3x^2 - 3)$ correctly and proceeded to give the correct result.

The most concise correct solutions were achieved when the first term was simplified to $\frac{1}{3(x-1)}$ as this made manipulation simpler. Some candidates, who factorised $(3x^2 - 3)$ correctly did not always cancel out the common factor of $(x + 1)$, and so made the combination of the two terms more complicated than necessary, often making careless errors as a result.

A significant proportion of candidates made a sign error usually by omitting a bracket around the subtracted term in their numerator. A few candidates incorrectly believed that $(3x + 1)$ could be factorised to give $3(x + 1)$.

3. Many candidates were able to obtain the correct answer in part (a) with a significant number of candidates making more than one attempt to arrive at the answer given in the question. Those candidates who attempted to combine all three terms at once or those who combined the first two terms and then combined the result with the third term were more successful in this part. Other candidates who started by trying to combine the second and third terms had problems dealing with the negative sign in front $\frac{2}{x+4}$ and usually added $\frac{2}{(x+4)}$ to $\frac{x-8}{(x-2)(x+4)}$ before combining the result with 1. It was pleasing to see that very few candidates used $(x + 4)^2(x - 2)$as their common denominator when combining all three terms.

In part (b), most candidates were able to apply the quotient rule correctly but a number of candidates failed to use brackets properly in the numerator and then found some difficultly in arriving at the given answer.

In part (c), many candidates were able to equate the numerator to the denominator of the given fraction and many of these candidates went onto form a quadratic in e^x which they usually solved. A significant number of candidates either failed to spot the quadratic or expanded $(e^x - 2)^2$ and then took the natural logarithm of each term on both sides of their resulting equation.

In either or both of parts (b) and (c), some candidates wrote e^{x^2} in their working instead of e^{2x} Such candidates usually lost the final accuracy mark in part (b) and the first accuracy mark in part (c).

4. This type of question has been set quite frequently and the majority of candidates knew the method well. Most approached the question in the conventional way by expressing the fractions with the common denominator $(x - 3)(x + 1)$. This question can, however, be made simpler by cancelling down the first fraction by $(x + 1)$, obtaining $\frac{2x+2}{x^2-2x-3} = \frac{2(x+1)}{(x+3)(x+1)} = \frac{2}{x-3}$.

Those who used the commoner method often had difficulties with the numerator of the combined fraction, not recognising that $-x^2 + 1 = 1 - x^2 = (1 - x)(1 + x)$ can be used to simplify this fraction. If part (a) was completed correctly, part (b) was almost invariably correct. It was possible to gain full marks in part (b) from unsimplified fractions in part (a), but this was rarely achieved.

5. A large number of candidates did not find this a friendly start to the paper, with quite a high proportion attempting the question more than once. There were many who dividing by $x^2 - 1$, showed insufficient knowledge of the method, stopping their long division before the final subtraction. Those getting as far as a linear remainder usually obtained the correct values of a and c, but the remainder was often incorrect. Errors often arose when not using the strategy of replacing $2x^4 - 3x^2 + x + 1$ by $2x^4 + 0x^3 - 3x^2 + x + 1$ and $x^2 - 1$ by $x^2 + 0x - 1$. It was not unusual to see candidates who completed long division correctly but who then, apparently not recognising the relevance of this to the question, went on to try other methods.

Most of those who used methods of equating coefficients and substituting values found 5 independent equations but completely correct solutions using these methods were uncommon. Very few decomposed the numerator and, generally, these appeared to be strong candidates. Of those who attempted to divide first by $x + 1$, and then by $x - 1$, few were able to deal with the remainders in a correct way.

6. Part (a) was very well done, the great majority of candidates gaining full marks. Part (b), however, proved very demanding and there were many who had no idea what is required to show a general algebraic result. It was common to see candidates, both here and in part (c), who substituted into the expression a number of isolated values of x, noted that they were all positive, and concluded the general result. Those who did complete the square correctly, obtaining $x^2 + x + 1 = \left(x + \frac{1}{2}\right)^2 + \frac{3}{4}$, did not always explain the relevance of this to the required result. Many tried calculus but, to complete the proof this way, it was necessary to show that $\left(-\frac{1}{2}, \frac{3}{4}\right)$,was a minimum and this was rarely seen. Those who tried to solve $x^2 + x + 1 = 0$ or just calculated the discriminant often correctly concluded that the graph did not cross the axis but, to complete this proof, it was necessary to use the fact that $x^2 + x + 1$ has a positive coefficient if x^2 and this was not often seen. A suitable diagram was accepted here as a sufficient supporting argument. A correct argument for part (c) was often given by those who were unable to tackle part (b) successfully and this was allowed the mark.

7. For many candidates this was a successful start to the paper with completely correct solutions being the norm. A few candidates used long division in part (a) but did not often succeed using this method. In part (b) candidates who chose a denominator other than the lowest common denominator often failed to gain the final two marks because they did not factorise the resulting cubic in the numerator. Some candidates ignored the hint given in the phrase 'hence or otherwise' and repeated the work they had already completed in part (a).

8. Pure Mathematics P2

Those candidates who factorised the two unfactorised terms and simplified before attempting to add their fractions were at a clear advantage here. There were a lot who did follow this route, but the alternative resulting in a numerator of $2x^3 + 5x^2 - 9x - 18$ was very popular. Many of these candidates never actually reached this expression due to a sign error in multiplying out their brackets at the previous stage resulting in $2x^3 + 5x^2 - 9x + 18$. Some candidates did not attempt to simplify the expression any further than this. There was quite a lot of correct factorization, some of it appearing with very little evidence of any working. Those candidates who achieved a factor of $(x+\frac{3}{2})$ with no obvious working were presumably working backwards from roots found using their calculators.

Almost all candidates started by looking for a common denominator in the early stages of their working. However, there were a small number who started by trying to split the original fractions into partial fractions. All but one of these candidates failed to appreciate that the first of the two fractions is top heavy so they should have been expecting a purely numerical term in addition to their fractions. Although this approach can sometimes produce a quick and simple solution to the task this was not the case on this occasion.

Core Mathematics C3

Those who realised that $\dfrac{2x^2 + 3x}{(2x+3)(x-2)}$ could be simplified to $\dfrac{x}{x-2}$ and then put the fractions over a common denominator usually completed the question quickly and full marks were very common. Those who put the fractions as they stood over a common denominator, if they work correctly, obtained the cubic term $2x^3 + 5x^2 - 9x - 18$ in the numerator and factorising this proved beyond many candidates. Those who could factorise the cubic often wasted valuable time.

9. Pure Mathematics P2

This question brought out the worst in some candidates' algebra.

(a) Despite a lot of fudging to get to the given answer, many candidates completed this successfully, although often using inefficient methods. Simply cross-multiplying the denominators led to a long quadratic numerator and a cubic denominator, which many then tidied up and factorised correctly. Most errors occurred when simplifying the numerator - quite a few candidates made a sign mistake when expanding the brackets and then tried to fudge the answer. Some candidates split the first part of the question into partial fractions, and then the required answer fell out very nicely.

(b) Some candidates confused the inverse function with the first derivative or with the reciprocal of the function. Both re-arrangement and flow-chart methods were seen. The most common errors were a sign error in the final answer, or not completing the working to express the inverse function as a function of x.

(c) A few candidates combined the functions in the wrong order, but many made a correct initial step. Some candidates made the question more difficult than necessary by reverting to the unsimplified version of f(x). A surprising number of candidates did not simplify the denominator $x^2 + 5 - 1$ correctly. Popular alternatives were $x^2 - 4$ and $-x^2 - 5$. Many candidates lost the final mark for ignoring the possibility of x being -2. Another common error was to substitute $x = \frac{1}{4}$ into the combined function. Some candidates never actually stated the combined function, they worked the problem in two stages, firstly finding $f^{-1}\left(\frac{1}{4}\right)$, then solving g($x$) = 9.

Core Mathematics C3

(a) Candidates found this the easiest question to complete accurately. Some did not use the lowest common denominator but mostfactorised later. There was a certain amount of alteration of signs in order to obtain the given answer.

(b) The inverse function was generally found correctly, although some candidates confused $f^{-1}(x)$ with f' (x).

(c) fg(x) was worked in the correct order by the majority of candidates. A significant group omitted the second solution x = -2 in their answer, focussing incorrectly on the domain given in part a). Another error was to find fg(1/4) instead of solving fg(x) = ¼.

10. Well received and well answered by the vast majority of candidates, who produced neat and concise solutions. Most errors were made in part (b) where the solutions were given as either ± 2 or just $\sqrt{2}$.

11. Many candidates gave completely correct solutions to part (a). However candidates with a correct solution to (a) often failed to complete part (b). This did seem, generally, to be the result of misunderstanding, as the algebra is straightforward. It is sufficient to give the working $2-\frac{1}{x+2}=\frac{2(x+2)-1}{x+2}=\frac{2x+3}{x+2}$ and note that this is the answer to part (a). Part (c) proved very demanding and many ignored that they were given that T_1 is a translation or did not know how to specify a translation.

12. This very friendly starter was gratefully received by the vast majority of candidates who often scored full marks. Factorising was usually correct, although some candidates lost marks for not fully factorising $2x^2 + 6x$, or for expressing $(x-5)^2$ as $(x-5)(x+5)$; in the latter case 3 marks could still be earned for the wrong answer of $\frac{2x}{x+5}$. Some candidates, however, got off to a very unproductive start by first multiplying out the expressions, which often took up much space, used precious time and usually gained no marks.

13. Most candidates found this a friendly question to start the paper. Weak algebra though meant that a number lost marks in the first two parts. A correct expression in part (a) was sometimes spoilt by false cancelling, and the most common error in part (b) was in multiplying by $(x + 2)$ producing $y(x + 2) = 2 + 3$. The stronger candidates usually score full marks on parts (a) and (b). In part (c) many failed to mention $x \in \mathbb{R}$ and others stated $x \neq -2$.

14. This question was well done and full marks were common. The only error seen at all frequently was to falsely 'cancel' $\frac{2(x+7)+13}{(x-3)(x+7)}$ to $\frac{2+13}{x-3}$.

15. Part (a) was a straightforward start to the paper with most candidates able to factorise and simplify perfectly. A few attempted to use long division but this often led to errors, and some weaker candidates simply cancelled the x^2 in the numerator and denominator of the expression.

The second part of this question turned out to be more testing, and it was clear that a number of candidates were not fully familiar with the rules of logarithms. Simply crossing out $\log_2$ was sometimes seen and a large number of candidates thought that $\log a - \log b \equiv \frac{\log a}{\log b}$. Those successfully reaching $\log_2\left(\frac{x+3}{x}\right)$ then had difficulty relating the base 2 with the 4 and common errors included $4^2 \log_2 4$, 4 log e. Even those who successfully arrived at $15x = 3$ were not yet home and dry, as a surprising number concluded that $x = 5$.

16. Well received and well answered by the vast majority of candidates, who produced neat and concise solutions. Considering this was not the easiest question of its type, there were fewer numerical or algebraic slips than in previous years. Even candidates who did not use the LCM in the denominator often picked up 3 or 4 marks, and candidates who “lost” the common factor of 2 in factorising the numerator in the final stages only lost one mark.

17. No Report available for this question.